The category $\mathcal{E}_{X/G}$ is "glued" out of the categories $\mathcal{E}_U/H$, where $U \subset X$ is an open affinoid subvariety $X$, and $H$ is an open subgroup of $G$ stabilizing $U$. (And some tech. cons.)

When $(U, H)$ is small, actually we have an equivalence

$$\mathcal{P}(U, -) : \mathcal{E}_U/H \to \mathcal{D}(U, H)$$

where $\mathcal{D}(U, H)$ is a certain completion of the sheaf group ring $D(U) \times H$, which will serve as a "local counterpart" to $D(G)$.

**Theorem** $\mathcal{D}(U, H)$ is Fréchet–Stein.

The construction of $\mathcal{E}_{X/G}$ from $\mathcal{D}(U, H)$ is analogous to the construction of $\text{coh}(Y)$ from $\{\text{coh}(\mathcal{O}(V)) : V \subset X \text{ is an affine open of } Y\}$ (where $Y$ is a locally Noetherian scheme).

**Theorem** $\mathcal{O}_G$ finite. Let $G$ be an affine algebraic group, f.t. over $L$. Let $g = \text{Lie}(G)$, assume $Z(g) = 0$. Let $g_K := g \otimes K$. Then isomorphism

$$D_{\text{an}}(G, K) \cong \mathcal{O}(g_K, G)$$

for any $G$, open subgroup of $G(L)$.

**Definition of $\mathcal{O}(g_K, G)$**

Fix a compact open $H \subset G$. Then

$$\mathcal{O}(g_K, G) = \mathcal{O}(g_H, H) \otimes_{KH} K[G]$$
as $U(g,H) = K[G] \otimes \text{bimodule}$.

(Kohlhaas)

Define a Lie lattice $L$ in $g$ is a f.g. $R$-submodule of $g$, s.t. $K L = g$
and s.t. $[K L] = L$.

Now $G(L)$ acts on $g$ by $L$-Lie adj. autos, hence so does $H \leq L \leq G(L)$.

Form $\widehat{U(L)} = \lim U(L)/\pi^k U(L)$ and $U(L)$.

If $H$ happens to stabilize $L$, then get an $H$-action on $U(L)$ by $R$-algebra
autos, so can form skew group rings

$\widehat{U(L)} \rtimes H$, and $\widehat{U(L)} \rtimes H$.

This is too big, so we shrink it.

Example $G = SL_2 \quad g = \mathfrak{sl}_2(L) = L e \oplus L f \oplus L h$.

$L = 0 e \oplus 0 f \oplus 0 h$.

$H = SL_2(O_2)$.

$\widehat{U(L)} = L < e, h, f > = \left\{ \sum_{ijk} a_{ijk} e^i h^j f^k : a_{ijk} \in L \land a_{ij0} \rightarrow 0 \right\}$.

(compare e.g. with $D_{\mathfrak{sl}_2}^{\mathfrak{sl}_2} (A^+_2) = L < x_1, x_2 >$)

Fact The adjoint action of a sufficiently small open subgroup $J \leq H$ can
be imitated as the exponential of the adjoint action of elements of $L$.

e.g. How does $\left( \begin{smallmatrix} e^p & 0 \\ 0 & e^p \end{smallmatrix} \right)$ act on $L < e, h, f >$? By

$\exp (p \text{ad}(h) : U(L) \rightarrow U(L))$.

So, for such $J$ can find a copy $\mathcal{B}(J)$ of $J$ inside $U(L)$ s.t.
\[ U(\mathfrak{g})_K \cong \bigotimes_{J < H} \left\{ h - \beta(h) : h \in J \right\} \]

**Definition 1.**
\[ U(\mathfrak{g})_K \times H := \left( U(\mathfrak{g})_K \times H \right)_{H/J} \]

**Fact 1.**
\[ U(\mathfrak{g})_K \times H \] is a crossed product of the Noetherian \( K \)-Banach algebra \( U(\mathfrak{g})_K \) and the finite group \( H/J \).

**Fact 2.** \( U(\mathfrak{g})_H \) is Frechet-Stallin.

**Definition of \( \mathcal{B}(U/H) \)**

Similarly:
\[ \mathcal{B}(U/H) = \lim_{(J, N)} U(\mathfrak{g})_K \times \text{Ann} H \]

where \( L \subset \text{Der}_K(O(U)) \) is a Lie lattice.

**Fact.** Given \( L \), you can find a copy \( \beta(N) \) of sufficiently small \( N \leq H \) in \( U(\mathfrak{g})_K \), st.

\[ \text{Hence, } N, \text{ } H \in U(\mathfrak{g})_K, \]

\[ h \cdot x = \beta(h) x \beta(h)^{-1} \]

**Example.**
\[ U = D^1 = \mathbb{C} \cdot K \langle x \rangle \]
\[ \partial(D(U)) = K \langle x \rangle \partial x \]
\[ \text{Der}(O(U)) = K \langle x \rangle \partial x \]
\[ L := R \langle x \rangle \partial x \quad \text{is an example} \]
of a Lie lattice in $K\langle x, \theta x \rangle$.

$$U(L) = \mathfrak{gl}(L) \otimes \mathbb{R}, \quad \mathfrak{h} = \mathfrak{gl}(L)$$

In general, for $L$ of e.g. projective as an $A$-module,

$$U(L) \cong \text{Sym}^A(L) \text{ as an } (A, \mathbb{A})\text{-module}$$

$$U(L)_K = K\langle x, \theta x \rangle \cong \mathbb{D}_0^\oplus (A^\times \mathbb{A})$$

For the natural Möbius action of $SL_2$ on $\mathbb{P}^1$, hence of $SL_2(\mathbb{O})$ on $(\mathbb{P}^1)_K$ does not preserve $\mathbb{D}$, but the first congruence subgroup

$$SL_2(q) = \ker(SL_2(q) \to SL_2(\mathbb{K}))$$

does.

And e.g. the (functorial) action of $(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix})$ on $K\langle x, \theta x \rangle$ is given by $\exp(p \text{ ad}(\theta x))$, also $(\begin{smallmatrix} 1 & 0 \\ p & 1 \end{smallmatrix})$ acts by $\exp(p \text{ ad}(-x^2 \theta x))$

Then we similarly prove $\mathcal{D}(U, \mathbb{A})$ is Fréchet–Stein.