Workshop on non-archimedean geometry

Padova, April 28-30, 2014

1 Fabrizio Andreatta

Title: *Instances of integral p-adic Hodge theory*

Abstract: We provide Hodge-Tate and crystalline integral comparison morphsims for abelian varieties admitting canonical subgroup. This gives new integral structures on Hodge and de Rham cohomology of the given abelian varieties. Time permitting we present applications (nearly holomorphic overconvergent modular forms, construction and properties of eigenvarieties).

2 Oren Ben-Bassat

Title: *Non-Archimedean Analytic Geometry as Relative Algebraic Geometry*

Abstract: I will review symmetric monoidal categories and explain how one can work with "algebras and modules" in such a category. Toën, Vaquié, and Vezzosi promoted the study of algebraic geometry relative to a closed symmetric monoidal category. By considering the closed symmetric monoidal category of Banach spaces, we recover various aspects of Berkovich analytic geometry. The opposite category to commutative algebra objects in a closed symmetric monoidal category has a few different notion of a Zariski toplogy. We show that one of these notions agrees with the G-topology of Berkovich theory and embed Berkovich analytic geometry into these abstract versions of algebraic geometry. In our context, the quasi-abelian categories of Banach spaces or modules as developed by Schneiders and Prosmans are very helpful. This is joint work with Kobi Kremnizer.

3 Valentina Di Proietto

Title: *On the homotopy exact sequence for the logarithmic de Rham fundamental group*

Abstract:

Let K be a field of characteristic 0 and let $X^\times$ be a quasi-projective simple normal crossing log variety over the log point $K^\times$ associated to K. We construct a log de Rham version of the homotopy sequence

$$\pi_1(X^\times/K^\times) \to \pi_1(X^\times/K) \to \pi_1(K^\times/K) \to 1$$

and prove that it is exact. Moreover we show the injectivity of the first map for certain quotients of the groups. Our proofs are purely algebraic. This is a joint work with A. Shiho.

4 Elmar Grosse-Klönne

Title: *A partial generalization of Colmez' functor*

Abstract: In Colmez' work on the mod-p local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, an important ingredient is a certain functor from smooth admissible mod p representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ to mod p representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$. I want to discuss a partial generalization of this functor for more general split reductive groups G over $\mathbb{Q}_p$. This functor is defined for smooth admissible mod p representations V of $G(\mathbb{Q}_p)$, but (in its present state) it is sensitive only to the space of invariants of V under a pro-p-Iwahori subgroup. For $G = \mathrm{GL}_n$ it induces a bijection between the set of isomorphism classes of simple supersingular n-dimensional modules over the mod p pro-p-Iwahori Hecke algebra for $G(\mathbb{Q}_p)$, and the set of isomorphism classes of irreducible n-dimensional mod p-representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$.

5 Johannes Nicaise

Title: ***Weight functions on Berkovich spaces and the Minimal Model Program.***

Abstract: I will explain some connections between Berkovich spaces of degenerations of Calabi-Yau varieties and the Minimal Model Program in birational geometry. The central object in this theory is the so-called weight function on the Berkovich space. This function has some interesting properties that suggest that one can use it to contract the Berkovich space onto its canonical skeleton. I will also show how analogous properties of weight functions of hypersurface singularities yield a proof of a 1999 conjecture of Veys on poles of maximal order of motivic zeta functions. This is based on joint work with Mircea Mustaţă and Chenyang Xu.

6 Jerôme Poineau

Title: *Berkovich spaces over $\mathbb{Z}$*

Abstract: Although Berkovich spaces usually appear in a non-archimedean setting, their general definition actually allows arbitrary Banach rings as base rings, e.g. $\mathbb{Z}$ endowed with the usual absolute value. Over the latter, Berkovich spaces look like fibrations that contain complex analytic spaces as well as p-adic analytic spaces for every prime number p. It is possible to generalize the Weierstrass division theorem to this context and use it to investigate the local properties of the spaces. We deduce that the structure sheaf of a Berkovich space over $\mathbb{Z}$ behaves as expected: it is coherent and its stalks are excellent local rings.

7 Andrea Pulita

Title: *p-adic differential equations over Berkovich curves*

Abstract: We deal with locally free $\mathcal{O}_X$ modules $\mathcal{F}$ with connection over a Berkovich curve X. The radii of convergence of the Taylor solutions of $\mathcal{F}$ are not all equal, as it happens over the complex numbers where all Taylor solutions converge in the largest disk of definition of the equation. Associating to each point of X the family of radii of convergence of the solutions it is possible to define a function with values in $\mathbb{R}^n$ called convergence Newton polygon. Roughly speaking the slopes of the polygon are the logarithms of the radii. Since Dwork and Robba, continuing with Christol-Mebkhout, this have been the main tool to classify p-adic differential equations. Indeed, as firstly obtained by Dwork-Robba locally on X, if $\mathcal{F}$ has two solutions with different radii this corresponds to a decomposition of $\mathcal{F}$ itself.

Moreover, as showed in the work of Christol-Mebkhout, the slopes of these radii are closely related to the local index of $\mathcal{F}$, and to its p-adic irregularity. More recently F.Baldassarri obtained an intrinsic definition of the radii over a Berkovich curve X, and he conjectured the existence of analogous results of a global form over X, establishing a link between the de Rham cohomology of the curve and some graphs inside X called controlling graphs. Indeed in this more global setting there is a new geometrical invariant attached to $\mathcal{F}$: a graph inside the curve outside which the radii functions are locally constant. The behavior of the radii is determined by their restriction to this graph. From a global point of view a basic step consists in showing that these graphs are locally finite. This have been achieved recently by myself together with J.Poineau, then by K.Kedlaya, and another proof have been announced by F.Baldassarri. With this result we have obtained more recently a global versions of the decomposition theorems, and some geometrical criteria (based on the nature of the controlling graphs) to have finite dimensionality of de Rham cohomology, together with global Grothendieck-Ogg-Shafarevich formula. This follows the original expectations of F.Baldassarri. The talk will be quite informal, and of an introductory style.

8 Jacques Tilouine

Title: *Big image of Galois and congruence ideals*

Abstract: In a joint work with Hida, we consider representations of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ associated to Hida families on GSp_4 of "general type". For those, we define a "Galois level" ideal in a several variable Iwasawa algebra and we study its relation with congruence ideals.